Conclusion

It has been shown in the paper that a nonuniform coaxial transmission line having an isoperimetric sheath deformation for impedance matching in uniform lines or for use in microwave components can be designed. It has also been shown that such a configuration can yield an electrical performance corresponding to some of the commonly adopted laws of taper. The new structure is believed to have certain distinct mechanical and constructional advantages. A possible mechanical setup, with its associated control equipment, for the construction of such a line, is suggested. As this setup is only representative, it is expected that further refinements may entail a more economical and accurate construction.

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Prototypes for Use in Broadbanding Reflection Amplifiers*

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Summary—This paper tabulates, as functions of reflection gain and ripple, the element values of negative-resistance terminated, prototype, low-pass, lumped-element ladder networks of normalized impedance and bandwidth. (The values are calculated using known synthesis methods.) Next, it provides a technique for relating the characteristics of any actual narrow-band, negative-resistance device to the value of the prototype susceptive element adjacent to the negative resistance. When an actual negative-resistance device has been related to a prototype in this manner, the performance of the device with one, two or three additional cascaded resonators can be predicted from given graphs. This allows trade-offs among gain, ripple, and bandwidth, within limits. Finally, the predicted performance can be used with simple formulas and the table of prototype element values to design suitable resonators to broadband the actual amplifier. The tables and techniques of this paper are used successfully to broadband tunnel-diode, maser and parametric-amplifier circuits.

This paper allows the practical engineer to estimate the broadbanding potential of any given negative-resistance device and provides him with the proper element values to do so with only a few very simple calculations required.

Introduction

IN RECENT YEARS reflection-type, negative-resistance amplifiers have received considerable attention. This has come about as a result of the introduction of circulators and of solid-state devices that are capable of presenting a negative resistance under the proper conditions. Typically, such an amplifier might consist of a negative resistance and an associated resonating structure terminating one port of a circulator. Assuming an ideal circulator to which the load and generator resistances are matched, the mid-band gain of the amplifier is determined by the ratio of the load resistance to the negative resistance. The bandwidth of the amplifier depends on the values of these resistances and the slope parameter of the resonating structure. A considerable improvement in the bandwidth of such an amplifier can be achieved by appropriately placing one or more additional resonating structures between the circulator and the terminals of the negative resistance (with its own resonating structure). Matthaei[1] has shown how this can be done for varactor-diode parametric amplifiers, while Kyhl, McFarlane and Strandberg[2] have demonstrated the use of an additional cavity to broaden the bandwidth of the cavity maser. While these references consider broadbanding from the point of view of the specific negative-resistance device employed, this paper considers the broadbanding of a very simple prototype device and discusses how to relate this prototype to any particular negative-resistance device. The results are applicable to many kinds of negative-resistance devices, yet only to the extent that they can be reasonably related to the prototype.

Design relations for broadbanding ideal negative-resistance devices (capacitance and negative-resistance in parallel) have previously been given for both maxi-

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[1] G. L. Matthaei, "A study of the optimum design of wide-band parametric amplifiers and up-converters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, pp. 23–38; January, 1961.

[2] R. L. Kyhl, R. A. McFarlane, and M. W. P. Strandberg, "Negative L and C in solid-state masers," PROC. IRE, vol. 50, pp. 1608–1623; July, 1962.

mally flat[3–5] and equal-ripple[4,5] reflection responses. Also, ultimate limitations[5] on gain-bandwidth performance have been considered. This paper, however, provides actual element values for broadbanding the ideal negative-resistance device, and also gives a practical, simple method for relating actual nonideal devices to the ideal prototypes. This method will be given after the prototype has been presented.

The Negative-Resistance Prototypes

Low-pass, lumped-element, normalized prototypes are generally used in the design of filters and impedance-matching networks.[1,6–8] The problem of determining a prototype terminated in a negative resistance is closely related to the design of the impedance-matching network.[9] The negative-resistance prototypes to be shown are based on impedance-matching network prototypes having Tchebyscheff ripple in their passbands. The relations involved are discussed further in Appendix I.

Two negative-resistance prototypes and their reflection responses are shown in Fig. 1. For the conventions

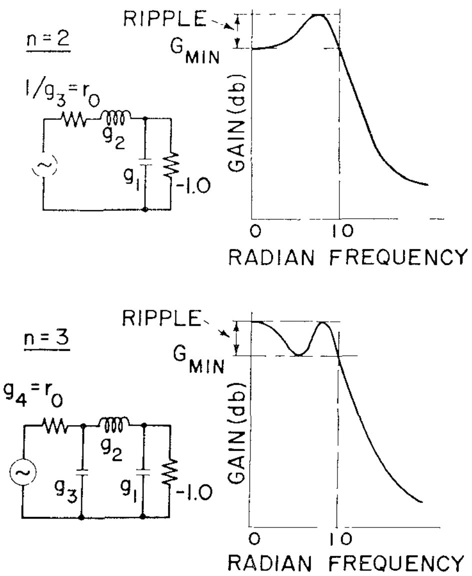

Fig. 1—Low-pass, negative-resistance prototypes.

[3] E. W. Sard, "Tunnel (Esaki) diode amplifiers with unusually large bandwidths," Proc. IRE (*Correspondence*), vol. 48, pp. 357–358; March, 1960.

[4] R. Aron, "Gain bandwidth relations in negative-resistance amplifiers," Proc. IRE (*Correspondence*), vol. 49, pp. 355–356; January, 1961. See also the comments by E. W. Sard following this letter.

[5] L. I. Smilen, "A Theory for Broadband Tunnel Diode Amplifiers," Polytech. Inst. of Brooklyn, N. Y., Rpt. No. PIBMRI-998-62, Contract No. AF-30(602)-2213; April 20, 1962.

[6] G. L. Matthaei, "Synthesis of Tchebycheff impedance-matching networks, filters, and interstages, IRE Trans. on Circuit Theory, vol. CT-3, pp. 163–172; September, 1956.

[7] L. Weinberg, "Network design by use of modern synthesis techniques and tables," *Proc. NEC*, Chicago, Ill., October 1–3, 1956, vol. 12, pp. 794–817.

[8] S. B. Cohn, "Direct-coupled resonator filters," Proc. IRE, vol. 45, pp. 187–196; February, 1957.

[9] G. L. Matthaei, *et al.*, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Res. Inst., Menlo Park, Calif., Final Report, Contract DA 36-039 SC-74862; January, 1961.

chosen here g_1 is a capacitance, g_2 an inductance, and so on, with $g_{n+1}=r_0$ (a resistance) for n odd, or $g_{n+1}=1/r_0$ (a conductance) for n even; n is the number of reactive elements only. The source resistance serves also as the termination for the reflected wave. Gain is defined as the ratio of the reflected power dissipated in the load resistance to the power available from the generator. This gain is normally denoted transducer gain.

Table I has actual values for the prototype elements tabulated against gain ($G_{\min}$) and ripple, both in decibels, for $n=2$, $n=3$, and $n=4$. Values are consistent with Fig. 1, requiring the negative resistance to have a magnitude of unity and the ripple band edge at radian frequency of unity.

Relating the Prototypes to De-Normalized, Lumped-Element Networks

The first step in relating a prototype to actual circuitry is to form a lumped-element network having the same gain and ripple as the prototype but with specified center-frequency, bandwidth and impedance level.

Dual-configuration, lumped-element, stable bandpass networks that can be related to the prototype, and the basic formulas relating the values of the network elements to the values of the prototype elements are shown in Fig. 2. In these formulas, R_0 and $-R$ are the source and negative resistances, respectively, when a series resonator is adjacent to the negative resistance. Similarly, R_0' and $-R'$ are used when a shunt resonator is next to the negative resistance. Series and shunt resonators alternate in the two circuits shown. The formulas differ, depending on whether the first resonator is to be series- or shunt-connected. Each resonator is resonant at the desired band center of radian frequency ω_0. For each resonator,

$$\omega_0^2 = 1/LC, \tag{1}$$

where L and C are the actual inductance and capacitance of the particular resonator being considered. The slope parameter x_j for the series resonator of subscript j is given in general by

$$x_j = \frac{\omega_0}{2} \frac{dX_j}{d\omega}\bigg|_{\omega=\omega_0} \tag{2}$$

where X_j is the reactance of the series resonator denoted by the subscript j. In the case of the lumped elements of Fig. 2, this equation simplifies to

$$x_j = \omega_0 L_j. \tag{3}$$

Similarly, for the shunt resonator of subscript k the slope parameter is given by

$$b_k = \frac{\omega_0}{2} \frac{dB_k}{d\omega}\bigg|_{\omega=\omega_0}, \tag{4}$$

TABLE I

Prototype Element Values: $n=2$, $n=3$, $n=4$

N=2

G MIN DB	RIP DB	G ONE	G TWO	G THREE
5.000000	0.050000	2.047029	0.090762	3.569771
5.000000	0.100000	2.373910	0.110079	3.569771
5.000000	0.200000	2.717309	0.134542	3.569771
5.000000	0.500000	3.142281	0.178296	3.569771
5.000000	1.000000	3.366818	0.224112	3.569771
5.000000	1.500000	3.423646	0.257965	3.569771
5.000000	2.000000	3.419884	0.285850	3.569771
5.000000	2.500000	3.389207	0.309936	3.569771
5.000000	3.000000	3.345843	0.331297	3.569771
8.000000	0.050000	1.123240	0.118139	2.322851
8.000000	0.100000	1.311097	0.142574	2.322851
8.000000	0.200000	1.515839	0.173040	2.322851
8.000000	0.500000	1.792064	0.226176	2.322851
8.000000	1.000000	1.972788	0.280055	2.322851
8.000000	1.500000	2.049030	0.318820	2.322851
8.000000	2.000000	2.083000	0.350145	2.322851
8.000000	2.500000	2.095366	0.376789	2.322851
8.000000	3.000000	2.095468	0.400110	2.322851
10.000000	0.050000	0.834764	0.129751	1.924951
10.000000	0.100000	0.976940	0.156244	1.924951
10.000000	0.200000	1.134010	0.189038	1.924951
10.000000	0.500000	1.352354	0.245540	1.924951
10.000000	1.000000	1.504684	0.301886	1.924951
10.000000	1.500000	1.576134	0.341831	1.924951
10.000000	2.000000	1.613766	0.373748	1.924951
10.000000	2.500000	1.633451	0.400644	1.924951
10.000000	3.000000	1.642492	0.423994	1.924951
12.000000	0.050000	0.649071	0.136702	1.670900
12.000000	0.100000	0.761136	0.164326	1.670900
12.000000	0.200000	0.886156	0.198319	1.670900
12.000000	0.500000	1.063612	0.256292	1.670900
12.000000	1.000000	1.192752	0.313285	1.670900
12.000000	1.500000	1.257234	0.353164	1.670900
12.000000	2.000000	1.294108	0.384704	1.670900
12.000000	2.500000	1.315985	0.411054	1.670900
12.000000	3.000000	1.328730	0.433757	1.670900
15.000000	0.050000	0.469574	0.139802	1.432581
15.000000	0.100000	0.551911	0.167702	1.432581
15.000000	0.200000	0.644755	0.201783	1.432581
15.000000	0.500000	0.779488	0.259173	1.432581
15.000000	1.000000	0.881797	0.314573	1.432581
15.000000	1.500000	0.935940	0.352678	1.432581
15.000000	2.000000	0.969092	0.382409	1.432581
15.000000	2.500000	0.990582	0.406960	1.432581
15.000000	3.000000	1.004796	0.427896	1.432581
20.000000	0.050000	0.298027	0.131194	1.222222
20.000000	0.100000	0.351245	0.156965	1.222222
20.000000	0.200000	0.411985	0.188153	1.222222
20.000000	0.500000	0.502288	0.239812	1.222222
20.000000	1.000000	0.573930	0.288488	1.222222
20.000000	1.500000	0.614000	0.321199	1.222222
20.000000	2.000000	0.640025	0.346245	1.222222
20.000000	2.500000	0.658075	0.366593	1.222222
20.000000	3.000000	0.671031	0.383694	1.222222
25.000000	0.050000	0.200869	0.113609	1.119170
25.000000	0.100000	0.237197	0.135667	1.119170
25.000000	0.200000	0.279000	0.162179	1.119170
25.000000	0.500000	0.342139	0.205553	1.119170
25.000000	1.000000	0.393617	0.245674	1.119170
25.000000	1.500000	0.423354	0.272154	1.119170
25.000000	2.000000	0.443296	0.292131	1.119170
25.000000	2.500000	0.457604	0.308152	1.119170
25.000000	3.000000	0.468262	0.321458	1.119170
30.000000	0.050000	0.140150	0.093649	1.065311
30.000000	0.100000	0.165732	0.111676	1.065311
30.000000	0.200000	0.195341	0.133228	1.065311
30.000000	0.500000	0.240559	0.168160	1.065311
30.000000	1.000000	0.278115	0.200021	1.065311
30.000000	1.500000	0.300269	0.220757	1.065311
30.000000	2.000000	0.315426	0.236220	1.065311
30.000000	2.500000	0.326523	0.248494	1.065311
30.000000	3.000000	0.334966	0.258593	1.065311

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N=3

G MIN DB	RIP DB	G ONE	G TWO	G THREE	G FOUR
5.000000	0.050000	3.331969	0.267590	1.480225	0.282907
5.000000	0.100000	3.609226	0.298983	1.712289	0.285538
5.000000	0.200000	3.863582	0.333932	1.985458	0.290802
5.000000	0.500000	4.109295	0.387519	2.401165	0.306513
5.000000	1.000000	4.161764	0.437078	2.718835	0.332345
5.000000	1.500000	4.111606	0.471951	2.875535	0.357693
5.000000	2.000000	4.032773	0.500368	2.959788	0.382527
5.000000	2.500000	3.945864	0.524956	3.004295	0.406821
5.000000	3.000000	3.858565	0.546898	3.024945	0.430554
8.000000	0.050000	1.937111	0.371582	0.853981	0.432964
8.000000	0.100000	2.111857	0.415407	0.984394	0.435286
8.000000	0.200000	2.281328	0.463897	1.138997	0.439927
8.000000	0.500000	2.472640	0.536446	1.382740	0.453732
8.000000	1.000000	2.561531	0.599948	1.587860	0.476278
8.000000	1.500000	2.576011	0.642064	1.705653	0.498219
8.000000	2.000000	2.564324	0.674885	1.782205	0.519542
8.000000	2.500000	2.541110	0.702323	1.834596	0.540238
8.000000	3.000000	2.512514	0.726152	1.871329	0.560301
10.000000	0.050000	1.486554	0.422623	0.662305	0.521697
10.000000	0.100000	1.625214	0.472625	0.761901	0.523777
10.000000	0.200000	1.762385	0.527874	0.879988	0.527930
10.000000	0.500000	1.924898	0.609885	1.067795	0.540258
10.000000	1.000000	2.012176	0.680178	1.230320	0.560313
10.000000	1.500000	2.038022	0.725586	1.327783	0.579736
10.000000	2.000000	2.041053	0.760176	1.394269	0.598523
10.000000	2.500000	2.033243	0.788539	1.442316	0.616675
10.000000	3.000000	2.019759	0.812767	1.478165	0.634194
12.000000	0.050000	1.190940	0.459836	0.540560	0.600417
12.000000	0.100000	1.304959	0.514395	0.620709	0.602244
12.000000	0.200000	1.419374	0.574643	0.715620	0.605890
12.000000	0.500000	1.559391	0.663636	0.867159	0.616693
12.000000	1.000000	1.641159	0.738814	1.000439	0.634205
12.000000	1.500000	1.671082	0.786424	1.082476	0.651093
12.000000	2.000000	1.681114	0.822034	1.140072	0.667362
12.000000	2.500000	1.681295	0.850757	1.183006	0.683019
12.000000	3.000000	1.676024	0.874936	1.216132	0.698073
15.000000	0.050000	0.899471	0.492406	0.424202	0.699589
15.000000	0.100000	0.988304	0.551044	0.485895	0.701047
15.000000	0.200000	1.078864	0.615791	0.558720	0.703954
15.000000	0.500000	1.193476	0.711024	0.675120	0.712550
15.000000	1.000000	1.265789	0.790323	0.778783	0.726424
15.000000	1.500000	1.296596	0.839485	0.844033	0.739733
15.000000	2.000000	1.310987	0.875496	0.891002	0.752488
15.000000	2.500000	1.316946	0.903975	0.926958	0.764703
15.000000	3.000000	1.318019	0.927511	0.955484	0.776393
20.000000	0.050000	0.612714	0.498173	0.312927	0.819179
20.000000	0.100000	0.675675	0.557797	0.357161	0.820118
20.000000	0.200000	0.741056	0.623680	0.409027	0.821989
20.000000	0.500000	0.826882	0.720318	0.491559	0.827505
20.000000	1.000000	0.885209	0.799773	0.565434	0.836360
20.000000	1.500000	0.913192	0.848009	0.612641	0.844802
20.000000	2.000000	0.928808	0.882565	0.647258	0.852843
20.000000	2.500000	0.937854	0.909293	0.674295	0.860500
20.000000	3.000000	0.942946	0.930906	0.696198	0.867787
25.000000	0.050000	0.443772	0.465624	0.247119	0.894128
25.000000	0.100000	0.490786	0.521566	0.281204	0.894700
25.000000	0.200000	0.540262	0.583450	0.320888	0.895840
25.000000	0.500000	0.606828	0.674200	0.383550	0.899194
25.000000	1.000000	0.654147	0.748423	0.439410	0.904561
25.000000	1.500000	0.678291	0.793017	0.475181	0.909657
25.000000	2.000000	0.692808	0.824582	0.501548	0.914494
25.000000	2.500000	0.702113	0.848687	0.522276	0.919083
25.000000	3.000000	0.708212	0.867927	0.539190	0.923436
30.000000	0.050000	0.333414	0.415205	0.201964	0.939052
30.000000	0.100000	0.369610	0.465235	0.229242	0.939389
30.000000	0.200000	0.408094	0.520639	0.260785	0.940060
30.000000	0.500000	0.460818	0.601944	0.310167	0.942034
30.000000	1.000000	0.499471	0.668329	0.353841	0.945186
30.000000	1.500000	0.519967	0.708021	0.381704	0.948172
30.000000	2.000000	0.532819	0.735944	0.402225	0.951000
30.000000	2.500000	0.541475	0.757121	0.418363	0.953677
30.000000	3.000000	0.547515	0.773901	0.431546	0.956211

N=4

G MIN DB	RIP DB	G ONE	G TWO	G THREE	G FOUR	G FIVE
5.000000	0.050000	4.067117	0.371272	3.823667	0.128714	3.569771
5.000000	0.100000	4.265940	0.397333	4.163568	0.148113	3.569771
5.000000	0.200000	4.430001	0.424924	4.499190	0.172666	3.569771
5.000000	0.500000	4.545024	0.466360	4.872849	0.216973	3.569771
5.000000	1.000000	4.498242	0.506094	5.019656	0.264118	3.569771
5.000000	1.500000	4.393585	0.535595	5.013455	0.299444	3.569771
5.000000	2.000000	4.278915	0.560596	4.957323	0.328835	3.569771
5.000000	2.500000	4.166251	0.582841	4.882189	0.354422	3.569771
5.000000	3.000000	4.059411	0.603106	4.800493	0.377260	3.569771
8.000000	0.050000	2.430347	0.534019	2.304947	0.177435	2.322851
8.000000	0.100000	2.563192	0.572286	2.518041	0.202431	2.322851
8.000000	0.200000	2.682265	0.612109	2.737978	0.233399	2.322851
8.000000	0.500000	2.796735	0.668730	3.014286	0.287590	2.322851
8.000000	1.000000	2.823660	0.717907	3.175971	0.343263	2.322851
8.000000	1.500000	2.802370	0.751468	3.232275	0.383893	2.322851
8.000000	2.000000	2.766292	0.778450	3.248102	0.417098	2.322851
8.000000	2.500000	2.725038	0.801621	3.244283	0.445613	2.322851
8.000000	3.000000	2.682429	0.822198	3.229971	0.470780	2.322851
10.000000	0.050000	1.894074	0.619039	1.828252	0.202109	1.924951
10.000000	0.100000	2.002472	0.663986	1.999482	0.229662	1.924951
10.000000	0.200000	2.102309	0.710558	2.178931	0.263443	1.924951
10.000000	0.500000	2.206311	0.775551	2.413300	0.321650	1.924951
10.000000	1.000000	2.245017	0.829690	2.564675	0.380348	1.924951
10.000000	1.500000	2.242173	0.865059	2.629503	0.422548	1.924951
10.000000	2.000000	2.225317	0.892597	2.659667	0.456673	1.924951
10.000000	2.500000	2.202606	0.915679	2.672077	0.485731	1.924950
10.000000	3.000000	2.177419	0.935793	2.674328	0.511195	1.924951
12.000000	0.050000	1.539498	0.685375	1.522322	0.220959	1.670900
12.000000	0.100000	1.630827	0.735758	1.666094	0.250258	1.670900
12.000000	0.200000	1.716537	0.787861	1.818352	0.285860	1.670900
12.000000	0.500000	1.810401	0.859716	2.022419	0.346399	1.670900
12.000000	1.000000	1.852870	0.917789	2.162431	0.406462	1.670900
12.000000	1.500000	1.859130	0.954403	2.228930	0.449070	1.670900
12.000000	2.000000	1.852538	0.982108	2.265234	0.483190	1.670900
12.000000	2.500000	1.840129	1.004799	2.285589	0.512017	1.670900
12.000000	3.000000	1.824882	1.024200	2.296465	0.537112	1.670900
15.000000	0.050000	1.187226	0.752175	1.227945	0.239468	1.432581
15.000000	0.100000	1.260775	0.808430	1.344798	0.270104	1.432581
15.000000	0.200000	1.331198	0.866578	1.469873	0.306909	1.432581
15.000000	0.500000	1.412175	0.945950	1.641896	0.368431	1.432581
15.000000	1.000000	1.454822	1.008145	1.766806	0.428181	1.432581
15.000000	1.500000	1.467311	1.045794	1.831425	0.469817	1.432581
15.000000	2.000000	1.468609	1.073275	1.870707	0.502725	1.432581
15.000000	2.500000	1.464518	1.095092	1.896230	0.530232	1.432581
15.000000	3.000000	1.457540	1.113247	1.913305	0.553962	1.432581
20.000000	0.050000	0.836990	0.791503	0.945424	0.249706	1.222222
20.000000	0.100000	0.891859	0.852249	1.035868	0.280095	1.222222
20.000000	0.200000	0.945608	0.915190	1.133698	0.316034	1.222222
20.000000	0.500000	1.010563	1.000585	1.271672	0.374707	1.222222
20.000000	1.000000	1.049369	1.065684	1.377209	0.430021	1.222222
20.000000	1.500000	1.064820	1.103448	1.435786	0.467610	1.222222
20.000000	2.000000	1.071251	1.129874	1.474223	0.496770	1.222222
20.000000	2.500000	1.073112	1.150033	1.501452	0.520778	1.222222
20.000000	3.000000	1.072355	1.166192	1.521607	0.541223	1.222222
25.000000	0.050000	0.627756	0.769122	0.778152	0.242959	1.119170
25.000000	0.100000	0.670809	0.829501	0.852613	0.271317	1.119170
25.000000	0.200000	0.713673	0.892317	0.933607	0.304422	1.119170
25.000000	0.500000	0.767156	0.977611	1.049401	0.357433	1.119170
25.000000	1.000000	0.801375	1.041965	1.140409	0.406203	1.119170
25.000000	1.500000	0.816766	1.078513	1.192674	0.438665	1.119170
25.000000	2.000000	0.824708	1.103482	1.228146	0.463465	1.119170
25.000000	2.500000	0.828774	1.122060	1.254155	0.483627	1.119170
25.000000	3.000000	0.830548	1.136584	1.274107	0.500612	1.119170
30.000000	0.050000	0.488870	0.713195	0.662674	0.227225	1.065311
30.000000	0.100000	0.523683	0.770292	0.725929	0.252817	1.065311
30.000000	0.200000	0.558781	0.829963	0.794933	0.282369	1.065311
30.000000	0.500000	0.603598	0.911298	0.894321	0.328930	1.065311
30.000000	1.000000	0.633575	0.972573	0.973601	0.370901	1.065311
30.000000	1.500000	0.647992	1.007074	1.019958	0.398356	1.065311
30.000000	2.000000	0.656150	1.030372	1.051959	0.419062	1.065311

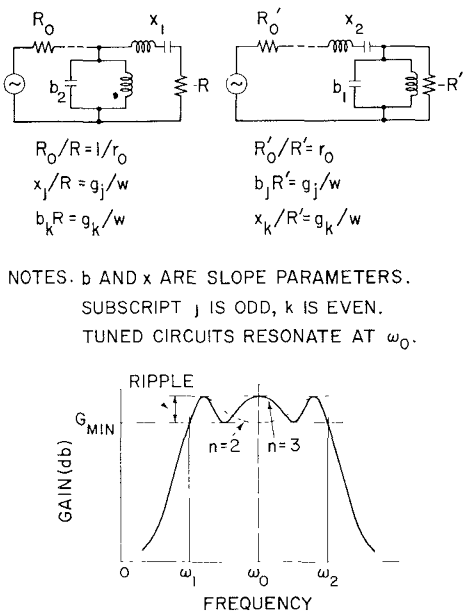


Fig. 2—Lumped-element, bandpass networks.

where B_k is the susceptance of the shunt resonator denoted by the subscript k. For lumped elements (4) becomes

$$b_k = \omega_0 C_k. \tag{5}$$

The factor w in the formulas of Fig. 2 is the fractional bandwidth defined by

$$w = \frac{\omega_2 - \omega_1}{\omega_0}, \tag{6}$$

where

$$\omega_0 = \sqrt{\omega_1 \omega_2}. \tag{7}$$

In (6) and (7) ω_2 and ω_1 are the edges of the equal-ripple amplification band, as shown in the response curve of Fig. 2.

Predicting Broadband Performance

An actual negative-resistance device has one or more reactive elements associated with it that determine the mimimum value of the slope parameter of the first resonator. The maximum bandwidth that can be achieved with a negative-resistance amplifier depends inversely on this slope parameter normalized against the negative resistance. It is, of course, this slope parameter that causes the initial limitation on bandwidth that the techniques of this paper are designed to alleviate. Therefore, designing the structure in a way that keeps the normalized slope parameter of the first resonator as small as possible is the single most important thing that can be done to maximize the bandwidth of the amplifier. For example, the physical circuitry supporting and connecting a diode cartridge in an amplifier should be as nearly lumped as possible and serve electrically only to resonate the unavoidable parasitic elements of the diode and its case. If the connecting circuitry adds to the parasitic elements of the diode or involves electrically long lengths of line, then the potential bandwidth of the amplifier has been reduced with or without an additional broadbanding resonator.

It will be assumed henceforth that the load resistance R_0 or R_0' can be achieved through a transformer if necessary and that the normalized slope parameter x_1/R or b_1/R' is fixed by the particular device and circuit arrangements to be used. (Determination of the device slope parameter is discussed in the next section.) Then the prototype element g_1 is found from

$$g_1 = w(x_1/R) \tag{8}$$

if the device resonator is assumed to be in series with the negative resistance, or

$$g_1 = w(b_1 R') \tag{9}$$

if the device resonator is assumed to be in shunt with the negative resistance. For a trial value of fractional bandwidth w, the combinations of gain and ripple that can be achieved may be found from Fig. 3 for $n=2$, Fig. 4 for $n=3$, or Fig. 5 for $n=4$. Thus, once the normalized slope parameter of the first resonator is specified, performance in terms of bandwidth, gain, ripple and number of resonators may be manipulated until a suitable combination is found for the amplifier. The generator impedance R_0 or R_0' is given by

$$R_0/R = 1/r_0 \tag{10}$$

if the device resonator is assumed to be in series with the negative resistance, or

$$R_0'/R' = r_0 \tag{11}$$

if the device resonator is assumed to be in parallel with the negative resistance. A curve of r_0 as a function of gain is given in Fig. 6.

When satisfactory performance has been predicted for a broadbanded reflection amplifier using a given device of known slope parameter, it is only necessary to relate the remaining prototype elements to the bandpass resonators of Fig. 2 and to relate these to actual circuitry. First, however, it is necessary to find the slope parameter of a given negative-resistance device.

Finding the Normalized Slope Parameter of the First Resonator

One way to find the normalized slope parameter of the first resonator is to calculate or measure directly the input impedance or admittance of the resonated negative-resistance device, in order to approximate the imaginary part by a straight line having the same average slope $\Delta X/\Delta\omega$ or $\Delta B/\Delta\omega$, and to approximate the real part by a constant $-R$ or $-1/R'$ over a desired frequency band. If this approach is practical, the slope parameter is given by (2) or (4), and (8) or (9) and may be used to relate x_1/R or b_1R' to the first prototype element, g_1. A way that is often simpler to find the normal-

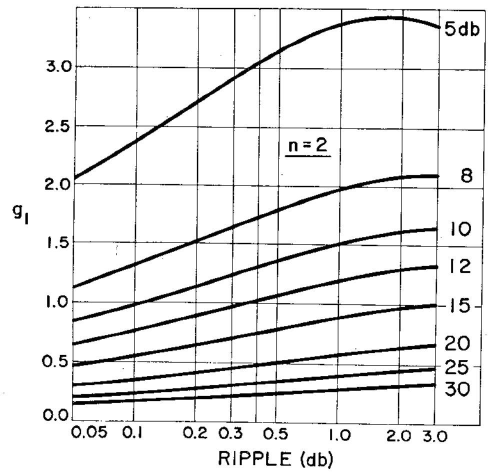


Fig. 3—First prototype element as a function of gain and ripple; $n=2$.

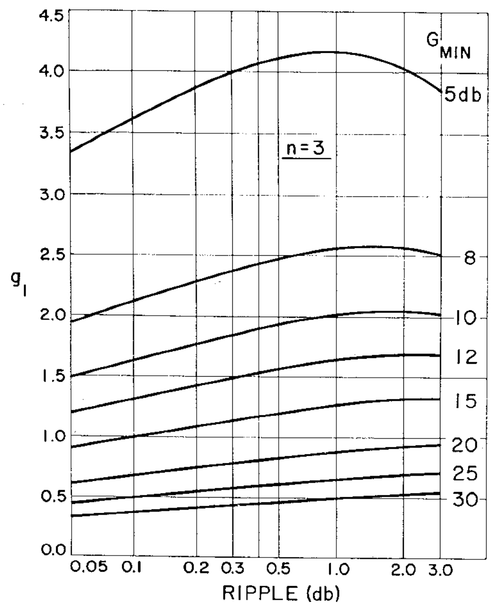


Fig. 4—First prototype element as a function of gain and ripple; $n=3$.

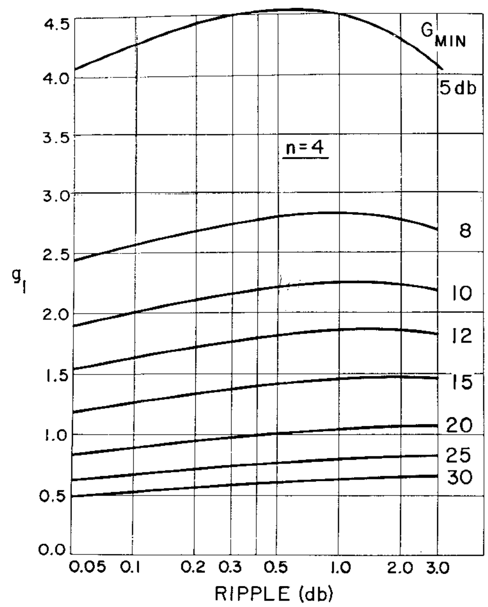


Fig. 5—First prototype element as a function of gain and ripple; $n=4$.

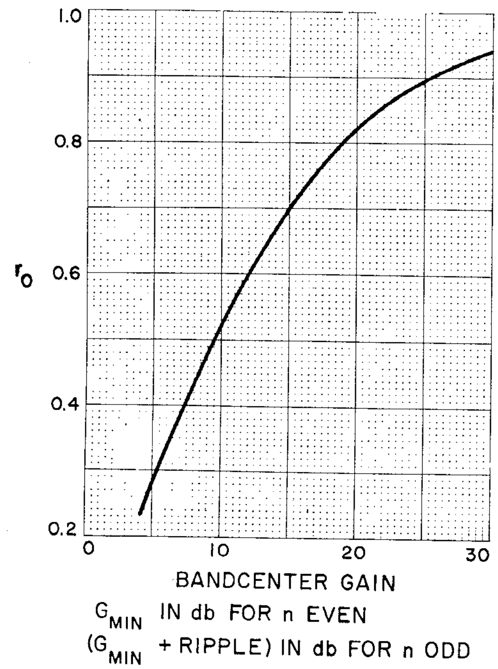


Fig. 6—Normalized generator resistance for prototypes.

ized slope parameter is by inference from a measured or calculated gain curve. That is, the basic negative-resistance device before broadbanding is made to operate (actually or on paper) at the desired frequency and approximately the desired center frequency gain; thereafter, the reflection gain in decibels is plotted as a function of frequency. The means of finding a simplified equivalent circuit having a gain curve approximating that of the actual device over a fairly wide frequency range is shown in Fig. 7. The normalized slope parameter of the simplified circuit is used as a basis to design the broadbanding network for the actual device.

As indicated in Fig. 7, the frequency difference $f''-f'$ between points at half the maximum decibel gain is found. This frequency difference is divided by the frequency f^0, at which frequency maximum gain occurs to give a normalized bandwidth U. Then a gain ratio K_0 is found by taking the antilog of the maximum gain in db. Finally, U and K_0 are substituted into the formula given in Fig. 7 (and derived in Appendix B) to find the normalized slope parameter x_1/R or b_1R', whichever is appropriate. This method was used for the examples in this paper.

Relating the Lumped-Element Bandpass Network to Actual Circuits

One of the first restrictions on the designer is the probability of his not realizing the circuitry in lumped elements. This problem has been handled in the case of filter design, and so will be discussed only briefly here.

Cohn[8] has given a lumped-element equivalent circuit for a half-wave long transmission-line resonator, and

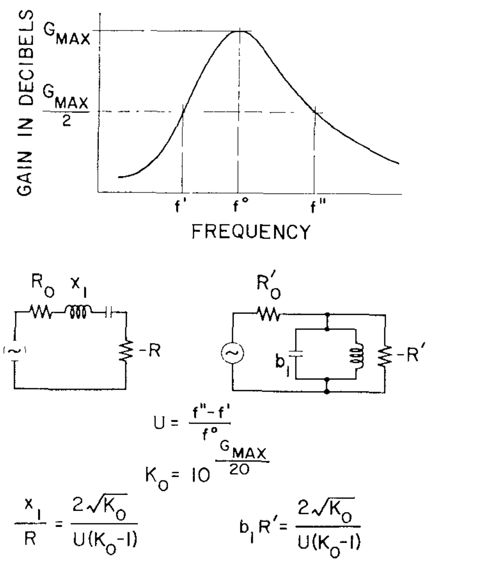


Fig. 7—Slope parameter related to gain curve.

has applied it, along with the concept of the impedance inverter (which he describes), to the design of waveguide and end-coupled strip-line filters. In a subsequent paper, Cohn[10] relates parallel-coupled transmission-line resonator filters to low-pass, lumped-element prototype filters. Equivalences between lumped-element resonators and transmission-line resonators determined on this basis are useful for bandwidths up to about twenty per cent. Matthaei[11] discusses the compensation used by Cohn[8] for the frequency sensitivity of impedance inverters and shows how direct-coupled filters with quarter-wave resonators can be related to filters with lumped elements.

Application to Actual Negative-Resistance Devices

The three negative-resistance devices to be considered are the tunnel-diode[12] amplifier, the cavity maser,[2] and the varactor-diode parametric amplifier.[1] All have complicated equivalent circuits between the negative-resistance element and the available terminals. Thus it is better to relate them to the prototype by way of the gain curves and information of Fig. 7 rather than by the use of admittance measurements. The simplest of these devices is the tunnel-diode amplifier.

To illustrate the determination of the slope parameter and the use of Table I, the steps involved in broadbanding a tunnel-diode amplifier are described at some length. First, it is assumed that the bias circuitry has negligible effect in the frequency range of interest. At the top of Fig. 8(a) is shown the equivalent circuit of a tunnel

[10] S. B. Cohn, "Parallel-coupled transmission-line resonator filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 223–231; April, 1958.

[11] G. L. Matthaei, "Direct-coupled, band-pass filters with $\lambda_0/4$ resonators," 1958 IRE National Convention Record, vol. 6, pt. 1, pp. 98–111.

[12] H. S. Sommers, Jr., "Tunnel diodes as high-frequency devices," Proc. IRE, vol. 47, pp. 1201–1206; July, 1959.

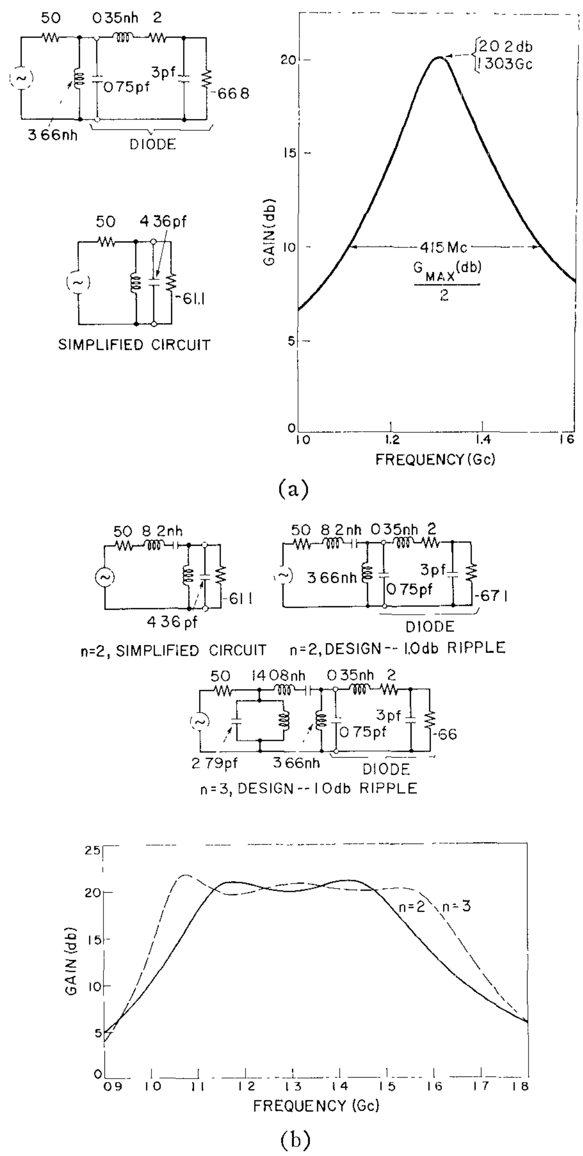


Fig. 8—(a) Tunnel diode example before broadbanding. (b) Tunnel diode examples; $n=2$, $n=3$.

diode resonated by a shunt inductance and connected to a generator. The generator resistance acts as the load for the amplified wave. The gain in decibels as a function of frequency for this amplifier is given by the curve of Fig. 8(a). This curve is obtained by direct calculation, using the circuit and element values shown at the top of Fig. 8(a). Comparing this curve with Fig. 7 and substituting $G_{\max}=20.2$ db, $f^\circ=1.303$ G_c and $f''-f'=0.415$ Gc into the formulas of Fig. 7, gives $U=0.3185$ and $K_0=10.2$. The diode here resonates in shunt, so a simplified bandpass circuit having very nearly the same gain curve would also be shunt resonated, as shown by the lower circuit of Fig. 8(a) and the right-hand circuit of Fig. 7. The numbers above are substituted into the formula for b_1R' of Fig. 7, giving $b_1R'=2.18$ for the simplified circuit of Fig. 8(b). Before assigning element values for the simplified circuit, it should be observed that 20.0-db gain in Table I is the nearest value to the gain of 20.2 db obtained in the actual circuit. Thus element values are based on 20.0-db gain without changing the value of the normalized slope parameter b_1R'. Eq. (11), also given in Fig. 2, shows that load (source) and negative resistances R_0' and $-R'$ are related by the term r_0, which can be taken from Fig. 6 for 20-db gain or from Table I by noticing that it must equal $1/g_{n+1}$ with n even for any ripple. (Also, r_0 would equal g_{n+1} for n odd at zero ripple, but this case is not covered in the tables.) It is found that $r_0=0.818$. Specifying a 50-ohm load sets the impedance level; and the element values for the simplified circuit of Fig. 8(a) result when the above derived numbers are used with (1), (5) and (11).

The gain curve for the simplified circuit lies too close along the curve for the diode circuit to allow the two curves to be shown separately for the frequency range on the graph of Fig. 8(a). This circuit can be directly related to the circuit of Fig. 2 and the prototypes of Fig. 1, and so can be broadbanded using Table I. For the case of one additional resonator ($n=2$) and specifying 20-db gain and 1.0-db ripple, Fig. 3 and Table I require that the first prototype element g have a value of 0.574; then, (8) (also given on Fig. 2) predicts a per-unit bandwidth w of 0.263, which represents a 342-Mc band centered at 1.3 Gc. Now the other element value $g_2 = 0.288$ is taken from Table I. This gives the value of 67 ohms for the series-resonator slope parameter x_2 when used in a formula from Fig. 2 along with the above found values of bandwidth w and negative resistance $-R'$. At the 1.3-Gc center frequency, an inductance of 8.2 nh is required [in accordance with (3)] and a capacitance to resonate with it. Using the element values found, the response of the simplified circuit will be exactly as predicted. This new simplified circuit, now containing the additional resonator represented by x_2, is on the upper left in Fig. 8(b). The technique proposed herein requires that the same additional resonator be appended to the diode amplifier. This has been done in the circuit shown on the upper right of Fig. 8(b), which is the broadbanded amplifier for $n=2$. The gain response of this amplifier is given by the curve of Fig. 8(b) labeled $n=2$. The diode negative resistance has been adjusted to make the center-frequency gain 20 db rather than 20.2 db. It was found that the difference in the gain curves of the actual diode circuit for $n=2$ and the simplified circuit for $n=2$ was too small to allow the plotting of separate curves. A bandwidth of 342 Mc was predicted. One of 344 Mc was achieved (by calculation). A ripple of 1.0 db was predicted by the prototype. One of 1.1 db was found from calculation. It should be noted that the diode characterization includes the various parasitic elements, and that lumped elements are assumed for the resonator.

Also shown on Fig. 8(b) is a gain curve in which two additional resonators are used. The design for this tunnel-diode amplifier was based on the values in Table I for $n=3$ with $G_{\min}=20$ db and ripple $=1.0$ db. The predicted bandwidth was 540 Mc. The asymmetry in the $n=3$ curve results from the difference between the actual diode circuit and the pure resistance-capacitance combination assumed for the prototype. Often such asymmetry can be corrected to some extent by detuning the resonator nearest the generator. To illustrate this possibility, Fig. 9 shows an amplifier design using the same diode with three additional resonators ($n=4$) as designed originally, and with the resonator nearest the generator having its resonant frequency raised by 10 Mc. The original design predicted a 20-db gain, a 1.0-db ripple and about a 625-Mc bandwidth. The performance actually achieved (on the computer) before attempting correction had a gain variation of about 3.6 db over the same 585-Mc bandwidth for which a variation of about 2 db was realized after touching up the last resonator.

The cavity maser equivalent circuit[2] is less accurately represented by the prototype than is that of the tunnel diode; but, the maser does have the virtue of yielding greater bandwidth when broadbanded than would be predicted from the prototype. Material linewidths, magnetic Q's and frequencies for the examples below are the same as used for some examples in Kyhl, *et al.*,[2] so that their theoretical results are comparable to the results below. Also, as in Kyhl, *et al.*,[2] cavity losses have not been considered. The above-mentioned bandwidth improvement is shown in Fig. 10 for a ruby cavity maser having a center frequency of 9 Gc. Here, curve (a) and circuit (a) refer to the maser before broadbanding. Curve (b) and circuit (b) refer to an idealized broadbanded ($n=2$) negative-resistance device whose element values are based on the gain curve for circuit (a). When the same resonator determined for (b) is used with (a), the curve and circuit (c) result. The decrease in ripple and increase in bandwidth of the actual maser circuit (c) over the idealized circuit (b) is clearly shown in Fig. 10.

The results of broadbanding another maser circuit and a table showing the difference between designed and realized performance are shown in Fig. 11. It should be noted that for the $n=3$ case the realized ripple is greater than the design value, but typically, the bandwidth is greater than that predicted. Justification for the increased bandwidth is discussed in Kyhl, *et al.*[2]

The last example is that for a nondegenerate varactor-diode parametric amplifier designed to operate from 7.5 to 8.5 Gc. In general, the parametric amplifier circuit is a very poor approximation to the simple lumped-resonator and constant negative-resistance circuit assumed from the prototype. Yet even in this case the technique and table for determining the resonator slope parameter can be used to advantage by giving results accurately enough to eliminate order-of-magnitude guesswork and to reduce cut-and-try to one or no cuts. The design of the paramp shown in Fig. 12 assumes the

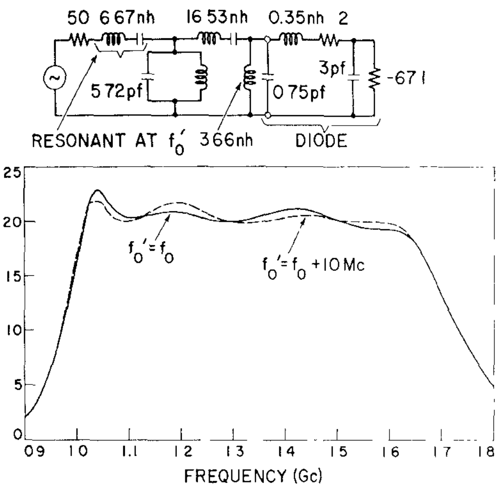


Fig. 9—Tunnel diode examples; $n=4$.

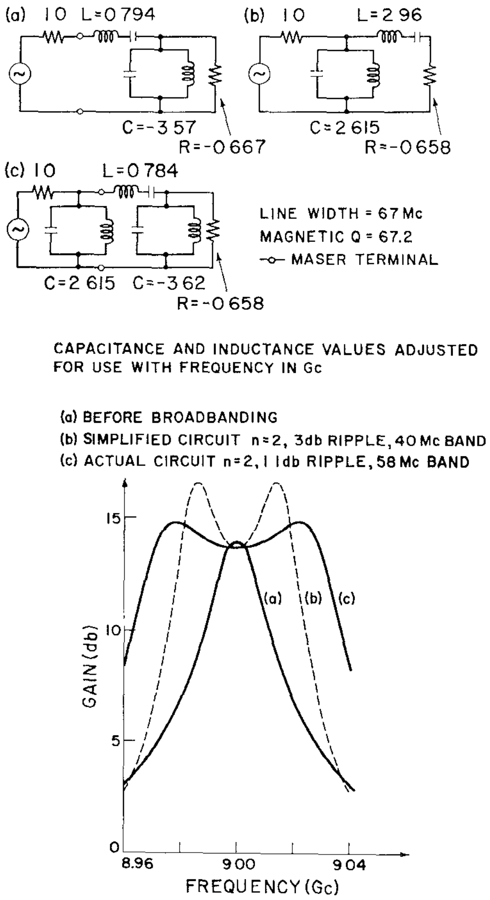
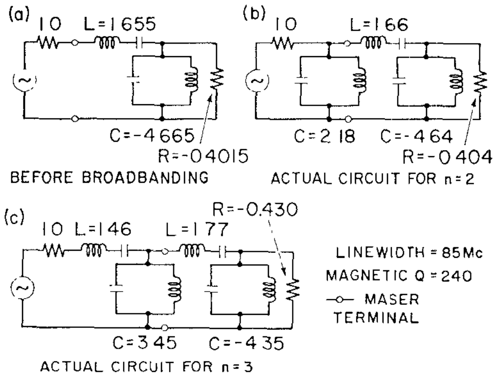


Fig. 10—Maser example showing bandwidth increase.



BROADBAND CIRCUIT		DESIGN BW (Mc)	DESIGN RIPPLE (db)	REALIZED BW (Mc)	REALIZED RIPPLE (db)
(b)	n=2	40	0 50	44	0 2
(c)	n=3	40	0 05	6 1	0 4

CAPACITANCE AND INDUCTANCE VALUES ADJUSTED FOR USE WITH FREQUENCY IN Gc

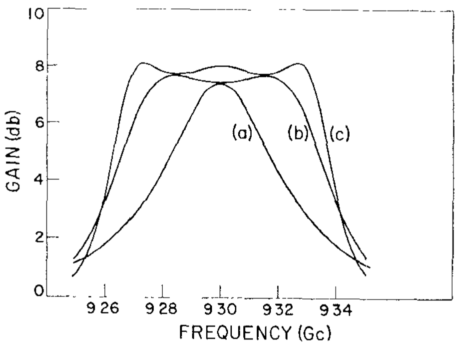


Fig. 11—Maser example; $n=2$, $n=3$.

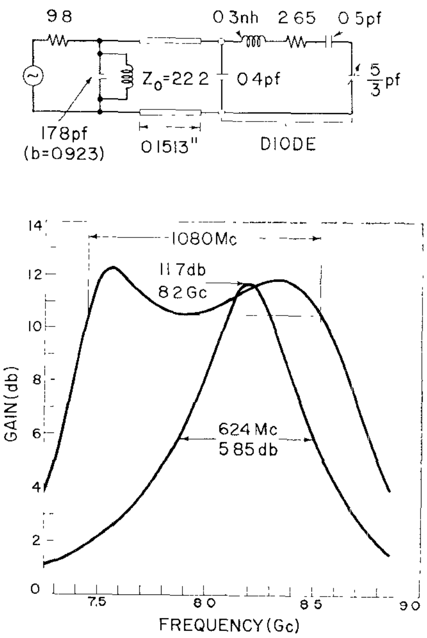


Fig. 12—Parametric amplifier example.

lumped shunt resonator to be a short circuit at the idler frequency. The transmission-line length was chosen to be a quarter wavelength at the idler center frequency, which was about 19 Gc. Under these conditions, the diode parasitic elements were resonant at the idler frequency; so the diode resistance of 2.65 ohms was the idler load. Next, the characteristic impedance of the transmission line was chosen to present a real negative impedance to the generator at the signal frequency of about 8 Gc. The bell-shaped gain curve of Fig. 12 was found by calculation on an electronic computer when the effect of the shunt resonator was ignored at the signal frequency (the generator resistance was 10 ohms). This curve was used with Fig. 7 to find x_1/R for the simplified approximate circuit. Then the values of g_1 and g_2 were taken from Table I for $n=2$, using a 12-db gain and a 2-db ripple. The formulas from Fig. 2 allowed prediction of a 588-Mc bandwidth, using slope parameter $b_2=0.923$ for the shunt resonator.

To summarize, the predicted performance was a 12-db center-frequency gain, a 2.0-db ripple and a 588-Mc ripple bandwidth, whereas the performance achieved (by calculation) was a 10.5-db center-frequency gain, about a 1.5-db ripple (with slight retuning of the resonator) and a 1080-Mc ripple bandwidth. Increasing or decreasing the resonator slope parameter would have increased or decreased the ripple with little effect on gain or bandwidth until the ripple was intolerably large or lost altogether. The center frequency of the shunt resonator was changed from its design value of 8.20 Gc to 8.25 Gc for the wideband gain curve shown. Another megacycle or two would have balanced the peaks. The increase over predicted bandwidth results from a negative-slope susceptance coupled into the signal line by the time-varying capacitance. This tends to broaden the predicted bandwidth in the same way that negative L and C in the maser equivalent circuit do.

Appendix I

Derivation of Prototype Element Values

It is well known[4] that the magnitude of the reflection coefficient of the LC ladder network terminated in a negative resistance is the reciprocal of the magnitude of the reflection coefficient that network would have when terminated in a positive resistance of the same magnitude.

If ρ is the voltage-reflection coefficient of the positively terminated network, then the power-reflection gain G of the negatively terminated network is

$$G = \frac{1}{|\rho|^2}, \tag{12}$$

because the voltage-reflection gain of the network is the same as the magnitude of its reflection coefficient. An LC ladder network terminated in a positive resistance and designed to have a reflection coefficient that varies between a specified maximum value ρ_{max} and a specified minimum value ρ_{min} over a specified frequency band is called an impedance-matching network.[6]

For the positively terminated impedance-matching network, the ripple H in terms of the power transmitted to the termination is

$$H = \frac{1 - |\rho_{min}|^2}{1 - |\rho_{max}|^2}. \tag{13}$$

The ripple S of the negatively terminated network, in terms of the power gain, is

$$S = \frac{G_{max}}{G_{min}}, \tag{14}$$

where G_{max} and G_{min} are the maximum transducer power gains within the passband of the reflection amplifier. Using (12),

$$G_{max} = \frac{1}{|\rho_{min}|^2},$$

$$G_{min} = \frac{1}{|\rho_{max}|^2}. \tag{15}$$

Eqs. (12)–(15) relate the performance of the negative-resistance reflection amplifier to the performance of the same network, operating as an impedance-matching network terminated with a positive resistance whose magnitude is the same as that of the negative resistance of the amplifier.

Ripple S and gain G_{min} were specified for the negative-resistance prototype, and reflection coefficients $|\rho_{min}|$ and $|\rho_{max}|$ were calculated for the corresponding impedance-matching network using the above equations. Then, element values for the network were computed from Green's formulas,[13] using the notation of Matthaei, *et al.*[9] The procedure is as follows. First, H is calculated from (13). Second, the parameter d is found from

$$d = \sinh\left\{\frac{\sinh^{-1}\sqrt{\dfrac{1}{H-1}}}{n}\right\}. \tag{16}$$

In (16) n is the number of reactive elements chosen for the network. It is necessary to calculate

$$U_n(e) = \frac{U_n(d)}{G_{min}}, \tag{17}$$

[13] E. Green, "Amplitude-Frequency Characteristics of Ladder Networks," Marconi's Wireless Telegraph Co., Ltd., Chelmsford, Essex, England.

where[6]

$$\left.\begin{aligned} U_2(x) &= 2x^2 + 1 \\ U_3(x) &= \sqrt{x^2+1}\,(4x^2+1) \\ U_4(x) &= 8x^4 + 8x^2 + 1 \end{aligned}\right\}. \tag{18}$$

It is necessary to solve $U_n(e)$ and (18) for e which is substituted into the following equation:

$$\delta = \frac{d - e}{2 \sin \frac{\pi}{2n}}, \tag{19}$$

where δ is the decrement. Then the prototype element values are as follows.

$$\left.\begin{aligned} g_1 &= 1/\delta \\ g_j \big|_{j=2 \text{ to } n} &= \frac{1}{g_{j-1} k_{j-1,j}^2} \\ g_{n+1} &= \frac{1}{D\delta g_n} \end{aligned}\right\}, \tag{20}$$

where

$$D = \frac{d}{\delta \sin \frac{\pi}{2n}} - 1. \tag{21}$$

For $n=2$,

$$k_{12}^2 = \frac{1 + (1 + D^2)\delta^2}{2}. \tag{22}$$

For $n=3$,

$$\left.\begin{aligned} k_{12}^2 &= \frac{3}{8}\left[1 + \left(1 + \frac{D^2}{3}\right)\delta^2\right] \\ k_{23}^2 &= \frac{3}{8}\left[1 + \left(\frac{1}{3} + D^2\right)\delta^2\right] \end{aligned}\right\}. \tag{23}$$

For $n=4$,

$$\left.\begin{aligned} k_{12}^2 &= \frac{1}{2\sqrt{2}}\left[1 + \left(1 + \frac{8D^2}{\alpha^4}\right)\delta^2\right] \\ k_{23}^2 &= \frac{2}{\alpha^2}\left[1 + \frac{2}{\alpha^2}(1 + D^2)\delta^2\right] \\ k_{34}^2 &= \frac{1}{2\sqrt{2}}\left[1 + \left(\frac{8}{\alpha^4} + D^2\right)\delta^2\right] \end{aligned}\right\}, \tag{24}$$

in which

$$\alpha^2 = 2(2 + \sqrt{2}) = 6.8284. \tag{25}$$

Green's equations as given above conform to the notation of Matthaei, *et al.*,[9] when the prototype radian-frequency band edge and the terminating resistance have both been normalized to unity, except where certain errors of transcription have been corrected.

Appendix II

Derivation of Slope Parameter from Gain Curve

With reference to Fig. 7, the gain curve, the circuit on the left, and the formulas are related. The reflection voltage gain K is equal to the magnitude of the circuit reflection coefficient. Thus,

$$K = \left| \frac{R_0 + R - jx_1\Omega}{R_0 - R + jx_1\Omega} \right|, \tag{26}$$

where

$$\Omega = \frac{f}{f^0} - \frac{f^0}{f}. \tag{27}$$

In (27), f is frequency and f^0 is defined by the graph of Fig. 7.

The band-center voltage gain K_0 is

$$K_0 = \left(\frac{R_0 + R}{R_0 - R}\right). \tag{28}$$

It is desired to find the value of Ω for which the square of the gain is equal to the band-center gain. Denoting this value of Ω by U,

$$K^2 = K_0 = \left| \frac{R_0 + R - jx_1 U}{R_0 - R + jx_1 U} \right|^2. \tag{29}$$

Substitution of (28) into (29) yields, after some manipulation,

$$\left(U \frac{x_1}{R}\right)^2 = \frac{4K_0}{(K_0 - 1)^2}. \tag{30}$$

Conservation of bandwidth implicit in the transformation (27) requires that the positive value of U in (30), representing the low-pass, half-decibel-gain band edge, be the same as the normalized bandpass, half-decibel-gain bandwidth occurring between f' and f'' of Fig. 7. Thus,

$$U = \frac{f'' - f'}{f^0}. \tag{31}$$

Now (30) can be written to give the slope parameter

$$\frac{x_1}{R} = \frac{2\sqrt{K_0}}{U(K_0 - 1)}. \tag{32}$$

The other equation of Fig. 7 comes about from duality.

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